

Charged Black Holes in Massive Gravity's Rainbow

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Violation of Lorentz invariance in the high energy quantum gravity, motivates one to consider an energy dependent spacetime with massive deformation of standard general relativity. In this paper, we take into account an energy dependent metric in the context of a massive gravity model to obtain exact solutions. We investigate the geometry of the solutions and show that there is a curvature singularity at the origin ($r = 0$) which can be covered with an event horizon. We also calculate the conserved and thermodynamic quantities, which are fully reproduced by the analysis performed with the standard techniques. Finally, we examine the validity of the first law of thermodynamics. Next, we conduct a study regarding the positivity and negativity of total mass in de Sitter and anti de Sitter spacetime.

I. INTRODUCTION

It is proven that Einstein theory of the gravity is an effective one which is valid in IR limit while in UV regime, it fails to produce accurate results. This shortcoming requires modification in order to incorporate UV regime. One of the UV completion theories is considering an energy dependent spacetime, the so-called gravity's rainbow. In transition from Galilean kinematic to special relativity, an upper limit of velocity is considered which is the speed of light. Such limitation could be generalized to energy scale as well which could be Planck scale. This is called as doubly special relativity [1]. Generalization of this doubly special relativity to curved spacetime is gravity's rainbow [2]. On the other hand, if one considers the gravity as an emerging phenomena due quantum degrees of freedom, spacetime should describe with an energy dependant metric. Therefore, the spacetime is affected by a particle probing it and since this particle can acquire a range of energies, a rainbow of energy is built.

On the other hand, gravity's rainbow has specific properties which highlight its importance in recent studies. First of all, this theory enjoys a modification in its energy-momentum dispersion relation. Such modification in UV limit is observed in studies that are conducted in discrete spacetime, models based on string theory, spacetime foam, spin-network in loop quantum gravity (LQG), non-commutative geometry, and also ghost condensation. In addition, the observational evidences confirm that such modification could exist [3]. Second, it was pointed out by treatment of the horizon radius of black holes as radial coordinate in this theory, the usual uncertainty principle stands [4]. Third, it was shown that the black hole thermodynamics in presence of gravity's rainbow is modified. Such modification leads to results such as existence of remnant for black holes [5] which is proposed to be a solution to information paradox [6]. In addition, the remnant put limitations on formation of mini black holes in the LHC.

Recently, the theoretical aspects of the gravity's rainbow have been investigate in different contexts [7]; different classes of black holes in presence of different gauge fields have been studied in Refs. [8]. In addition, the hydrostatic equilibrium equation of stars and the effects of this generalization on compact stars have been investigated [9][10]. Also, wormhole solutions in presence of gravity's rainbow are obtained in [11]. The effects of rainbow functions on gravitational force are studied in Ref. [12].

The elementary motivation of considering gravity's rainbow comes from the violation of Lorentz invariance in the high energy regime. In addition, in this regime one may regard a massive deformation of standard general relativity to obtain a Lorentz-invariant theory of massive gravity. It was shown that the Lorentz-breaking mass term of the graviton leads to a physically viable ghost free model [13]. The graviton in general relativity are massless particle with 2 degrees of freedom. Since there are arguments regarding the existences of graviton as a massive particle, it is necessary to modify general relativity to incorporate such property. The first attempt for building such theory was done by Fierz-Pauli [14] which later was shown by Boulware-Deser that suffers the ghost instability in non-linear extension [15]. Later, de Rham, Gabadadze and Tolley (dRGT) proposed another theory of massive gravity [16], which was proven that Boulware-Deser ghost was absent in it [17]. dRGT massive gravity employs a reference metric for constructing mass terms. After that, Vegh used a singular metric for constructing a dRGT like theory [18]. In his

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theory, the graviton behaves like a lattice in specific limits and a Drude peak was observed. It was shown that for arbitrary singular metric the mentioned theory is ghost free and enjoys the stability [19]. In addition, the stability issue was addressed in Ref. [20] and it was proven to be stable.

In this paper, we regard charged black holes in presence of massive gravity's rainbow. We point out that in order to obtain consistent solutions, one should modify the metric of massive gravity to an energy dependent one. We also obtain exact solutions, conserved and thermodynamic quantities. Next, we check the validity of the first law of black hole thermodynamics.

II. BASIC EQUATIONS

The d -dimensional action of Einstein- Λ -massive gravity with a linear $U(1)$ gauge field is

$$\mathcal{I} = -\frac{1}{16\pi} \int d^d x \sqrt{-g} \left[\mathcal{R} - 2\Lambda - \mathcal{F} + m^2 \sum_i^4 c_i \mathcal{U}_i(g, f) \right], \quad (1)$$

where Λ is the negative cosmological constant, \mathcal{R} is the scalar curvature and f is a fixed symmetric tensor. Also, $\mathcal{F} = F_{\mu\nu} F^{\mu\nu}$ is the Maxwell invariant where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is electromagnetic tensor field and also A_μ is the gauge potential. It is notable that, in Eq. (1), c_i 's are constants and \mathcal{U}_i 's are symmetric polynomials of the eigenvalues of the $d \times d$ matrix $\mathcal{K}_\nu^\mu = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$ which can be written as

$$\mathcal{U}_1 = [\mathcal{K}], \quad (2)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2], \quad (3)$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \quad (4)$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]. \quad (5)$$

Variation of the action (1) with respect to the metric tensor ($g_{\mu\nu}$) and the Faraday tensor ($F_{\mu\nu}$), leads to following field equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + 2 \left(F_{\mu\rho} F_\nu^\rho - \frac{1}{4} g_{\mu\nu} \mathcal{F} \right) + m^2 \chi_{\mu\nu} = 0, \quad (6)$$

$$\partial_\mu (\sqrt{-g} F^{\mu\nu}) = 0, \quad (7)$$

in which $G_{\mu\nu}$ is the Einstein tensor and $\chi_{\mu\nu}$ is

$$\begin{aligned} \chi_{\mu\nu} = & -\frac{c_1}{2} (\mathcal{U}_1 g_{\mu\nu} - \mathcal{K}_{\mu\nu}) - \frac{c_2}{2} (\mathcal{U}_2 g_{\mu\nu} - 2\mathcal{U}_1 \mathcal{K}_{\mu\nu} + 2\mathcal{K}_{\mu\nu}^2) - \frac{c_3}{2} (\mathcal{U}_3 g_{\mu\nu} - 3\mathcal{U}_2 \mathcal{K}_{\mu\nu} \\ & + 6\mathcal{U}_1 \mathcal{K}_{\mu\nu}^2 - 6\mathcal{K}_{\mu\nu}^3) - \frac{c_4}{2} (\mathcal{U}_4 g_{\mu\nu} - 4\mathcal{U}_3 \mathcal{K}_{\mu\nu} + 12\mathcal{U}_2 \mathcal{K}_{\mu\nu}^2 - 24\mathcal{U}_1 \mathcal{K}_{\mu\nu}^3 + 24\mathcal{K}_{\mu\nu}^4). \end{aligned} \quad (8)$$

III. CHARGED BLACK HOLE SOLUTIONS IN MASSIVE GRAVITY'S RAINBOW

Here, we are going to obtain topological static charged black holes with anti-de Sitter asymptote in the massive gravity's rainbow. Therefore, we consider the metric of d -dimensional spacetime with the following form

$$ds^2 = -\frac{\psi(r)}{f^2(E)} dt^2 + \frac{1}{g^2(E)} \left[\frac{dr^2}{\psi(r)} + r^2 h_{ij} dx_i dx_j \right], \quad i, j = 1, 2, 3, \dots, n, \quad (9)$$

where $h_{ij} dx_i dx_j$ is an spacial line element with constant curvature $d_2 d_3 k$ and volume V_{d_2} , in which $d_i = d - i$. We should note that the constant k , which indicates the boundary of $t = \text{constant}$ and $r = \text{constant}$, can be a positive (elliptic), zero (flat) or negative (hyperbolic) constant curvature hypersurface.

We consider the ansatz metric where introduced in Ref. [21]

$$f_{\mu\nu} = \text{diag}(0, 0, c^2 h_{ij}), \quad (10)$$

where c is a positive constant. Here, we use the metric ansatz (10), and obtain the \mathcal{U}_i in the following forms [21]

$$\mathcal{U}_1 = \frac{d_2 c}{r}, \quad (11)$$

$$\mathcal{U}_2 = \frac{d_2 d_3 c^2}{r^2}, \quad (12)$$

$$\mathcal{U}_3 = \frac{d_2 d_3 d_4 c^3}{r^3}, \quad (13)$$

$$\mathcal{U}_4 = \frac{d_2 d_3 d_4 d_5 c^4}{r^4}. \quad (14)$$

Inserting the gauge potential ansatz $A_\mu = h(r)\delta_\mu^0$ in the Maxwell equation (7) and considering the metric (9), we obtain

$$h(r) = -\frac{q}{r^{d_3}}, \quad (15)$$

where q is an integration constant and is related to the electric charge parameter. Also, the Maxwell equation (7) implies that the nonzero component of electromagnetic field tensor in d -dimensions is given by

$$F_{tr} = \frac{q d_3}{r^{d_2}}. \quad (16)$$

Now, we are interested in obtaining the static black hole solutions for this gravity. In order to obtain the metric function $\psi(r)$, one may use the nonzero components of Eq. (6). We use the tt and $x_1 x_1$ components of the Eq. (6), which can be written in the following form

$$\begin{aligned} tt = & \frac{d_2}{2} \psi'(r) g^2(E) r^{d_3} + \frac{d_2 d_3 r^{d_4}}{2} (\psi(r) - k) g^2(E) + \Lambda r^{d_2} + h'(r) g^2(E) f^2(E) r^{d_2} \\ & - \frac{m^2 d_2}{2} [cc_1 r^{d_3} + d_3 c_2 c^2 r^{d_4} + d_3 d_4 c_3 c^3 r^{d_5} + d_3 d_4 d_5 c_4 c^4 r^{d_6}], \end{aligned} \quad (17)$$

$$\begin{aligned} x_1 x_1 = & 2d_3 \psi'(r) g^2(E) r^{d_3} + d_3 d_4 r^{d_4} (\psi(r) - k) g^2(E) + 2\Lambda r^{d_2} - 2h'(r) g^2(E) f^2(E) r^{d_2} \\ & - m^2 d_3 [cc_1 r^{d_3} + d_4 c_2 c^2 r^{d_4} + d_4 d_5 c_3 c^3 r^{d_5} + d_4 d_5 d_6 c_4 c^4 r^{d_6}], \end{aligned} \quad (18)$$

Now, we can obtain the metric function $\psi(r)$, by using the Eqs. (17) and (18). Therefore, we have

$$\begin{aligned} \psi(r) = & k - \frac{m_0}{r^{d_3}} - \frac{2\Lambda}{d_1 d_2 g^2(E)} r^2 + \frac{2d_3 q^2 f^2(E)}{d_2 r^{2d_3}} \\ & + \frac{m^2}{g^2(E)} \left\{ \frac{cc_1}{d_2} r + c^2 c_2 + \frac{d_3 c^3 c_3}{r} + \frac{d_3 d_4 c^4 c_4}{r^2} \right\}, \end{aligned} \quad (19)$$

where m_0 is an integration constant which is related to the total mass of this black hole. It is notable that, the obtained metric function (19), satisfy all components of the Eq. (6).

In order to investigate the geometrical structure of this solution, we first look for the essential singularity(ies). By calculations of the Ricci and Kretschmann scalars, we find

$$\lim_{r \rightarrow 0} R \rightarrow \infty, \quad (20)$$

$$\lim_{r \rightarrow \infty} R = \frac{2d}{d_2} \Lambda, \quad (21)$$

$$\lim_{r \rightarrow 0} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \rightarrow \infty, \quad (22)$$

$$\lim_{r \rightarrow \infty} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = \frac{8d}{d_1 d_2^2} \Lambda^2. \quad (23)$$

These scalars are finite for $r \neq 0$, and therefore, we conclude that there is only one curvature singularity located at the origin ($r = 0$). In addition, Eqs. (21) and (23) confirm that the asymptotic behavior of this solution is adS. It is notable that, in the absence of massive parameter ($m = 0$), the solution (19) reduces to a d -dimensional asymptotically adS topological rainbow charged black hole as

$$\psi(r) = k - \frac{m_0}{r^{d_3}} - \frac{2\Lambda}{d_1 d_2 g^2(E)} r^2 + \frac{2d_3 q^2 f^2(E)}{d_2 r^{2d_3}}. \quad (24)$$

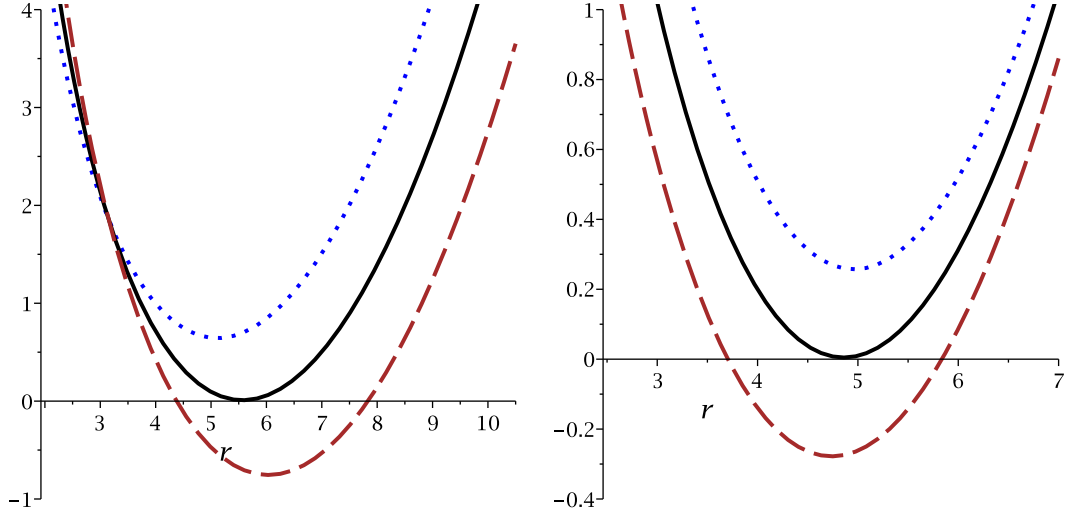


FIG. 1: $\psi(r)$ versus r for $g^2(E) = 0.9$, $f^2(E) = 1.1$, $\Lambda = -1$, $q = 1.4$, $m_0 = 10$, $c = 1.2$, $c_1 = -2$, $c_2 = 0.2$, $c_4 = 1.4$, $k = 1$, and $d = 5$.

Left diagram for $c_3 = 1.5$, $m = 1.30$ (dotted line), $m = 1.38$ (continues line) and $m = 1.45$ (dashed line).
Right diagram for $m = 1.3$, $c_3 = 1.20$ (dotted line), $c_2 = 1.01$ (continues line) and $c_2 = 0.80$ (dashed line).

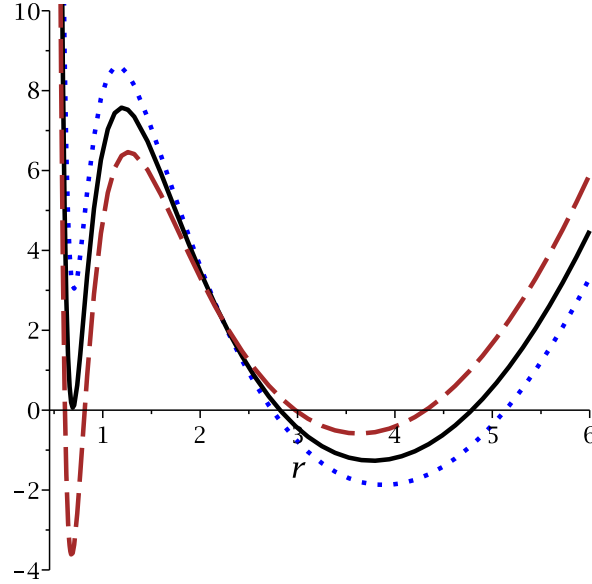


FIG. 2: $\psi(r)$ versus r for $g^2(E) = 1.1$, $f^2(E) = 1.1$, $\Lambda = -10$, $q = 1.4$, $m_0 = 35$, $c = 1.2$, $c_1 = -11$, $c_2 = 0.2$, $c_3 = 1.5$, $c_4 = 1.4$, $k = 1$, $d = 5$, $m = 1.370$ (dotted line), $m = 1.343$ (continues line) and $m = 1.310$ (dashed line).

On the other hand, considering $f(E) = g(E) = 1$, the solution (24) reduces to the Reissner-Nordström black hole in d -dimensions which is

$$\psi(r) = k - \frac{m_0}{r^{d_3}} - \frac{2\Lambda}{d_1 d_2} r^2 + \frac{2d_3 q^2}{d_2 r^{2d_3}}. \quad (25)$$

In order to study the effects of the massive term with rainbow functions on the metric, we plot some diagrams of $\psi(r)$ versus r (see Figs. 1 and 2).

As one can see, by considering specific values for different parameters, the metric function has different behaviors. In special case, massive charged solutions in the gravity's rainbow may be like Reissner-Nordström black holes (see Fig. 1 for more details). On the other hand, by adjusting some of the parameters, we encounter with interesting

behaviors. The solutions may have three or more horizons (see Fig. 2 for more details). As a result, we can state that, the existence of three or more roots for the metric function is due to the presence of massive gravity [22, 23].

IV. THERMODYNAMICS

Now, we should calculate the conserved and thermodynamics quantities of the solutions to check the first law of thermodynamics. We use the definition of Hawking temperature which is obtained through the concept of surface gravity on the outer horizon r_+ . We find

$$\begin{aligned} T &= \frac{g(E)}{4\pi f(E)} \psi'(r)|_{r=r_+} \\ &= \frac{kd_3g(E)}{4\pi f(E)r_+} - \frac{r_+\Lambda}{2\pi d_2f(E)g(E)} - \frac{d_3^2q^2g(E)f(E)}{2\pi d_2r^{2d_5/2}} \\ &\quad + \frac{m^2}{4\pi f(E)g(E)r_+^3} (d_3d_4d_5c_4c^4 + d_3d_4c_3c^3r_+ + d_3c_2c^2r_+^2 + cc_1r_+^2). \end{aligned} \quad (26)$$

In order to obtain the entropy of the black holes, one can employ the area law of the black holes. It is a matter of calculation to show that entropy has the following form [24]

$$S = \frac{V_{d_2}}{4} \left(\frac{r_+}{g(E)} \right)^{d_2}. \quad (27)$$

The electric charge, Q , can be found by calculating the flux of the electric field at infinity, yielding

$$Q = \frac{d_3V_{d_2}f(E)}{4\pi g^{d_3}(E)} q. \quad (28)$$

It was shown that by using the Hamiltonian approach, one can find the mass M of the black hole for massive gravity's rainbow as

$$M = \frac{d_2V_{d_2}}{16\pi f(E)g^{d_3}(E)} m_0, \quad (29)$$

in which by evaluating metric function on horizon ($\psi(r=r_+) = 0$), one can obtain geometrical mass and inserting it in Eq. (29), it will lead to

$$\begin{aligned} M &= \frac{d_2V_{d_2}}{16\pi f(E)g^{d_3}(E)} \left(kr_+^{d_3} - \frac{2r_+^{d_1}}{d_1d_2g^2(E)}\Lambda + \frac{2d_3q^2f^2(E)}{d_2r_+^{d_3}} \right. \\ &\quad \left. + \frac{m^2r_+^{d_5}}{d_2g^2(E)} [d_2d_3d_4c_4c^4 + d_2d_3c_3c^3r_+ + d_2c_2c^2r_+^2 + cc_1r_+^2] \right). \end{aligned} \quad (30)$$

It is notable that, U is the electric potential, where is defined in the following form

$$U = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r \rightarrow r_+} = \frac{q}{r_+^{d_3}}. \quad (31)$$

Having conserved and thermodynamic quantities at hand, we are in a position to check the validity of the first law of thermodynamics for our solutions. It is easy to show that by using thermodynamic quantities such as entropy (27), charge (28) and mass (29), with the first law of black hole thermodynamics

$$dM = TdS + UdQ, \quad (32)$$

we define the intensive parameters conjugate to S and Q . These quantities are the temperature and the electric potential

$$T = \left(\frac{\partial M}{\partial r_+} \right)_q \left(\frac{\partial r_+}{\partial S} \right)_q \quad \& \quad U = \left(\frac{\partial M}{\partial q} \right)_{r_+} \left(\frac{\partial q}{\partial Q} \right)_{r_+}, \quad (33)$$

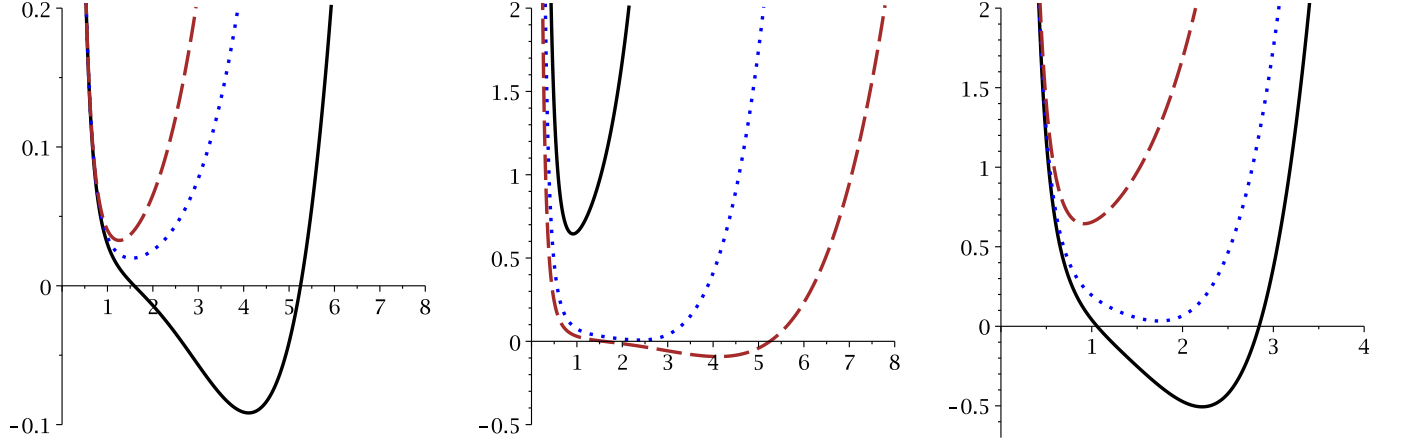


FIG. 3: $\psi(r)$ versus r for $q = 1$, $c = c_1 = c_2 = c_3 = c_4 = 0.1$, $d = 6$ and $\Lambda = -1$.

Left diagram for $g(E) = f(E) = 0.9$, $m = 2$, $k = -1$ (continues line), $k = 0$ (dotted line) and $k = 1$ (dashed line).

Middle diagram for $k = -1$, $m = 2$, $g(E) = f(E) = 0.9$ (continues line), $g(E) = f(E) = 1.45$ (dotted line) and $g(E) = f(E) = 2$ (dashed line).

Right diagram for $k = -1$, $g(E) = f(E) = 0.9$, $m = 0$ (continues line), $m = 11$ (dotted line) and $m = 2$ (dashed line).

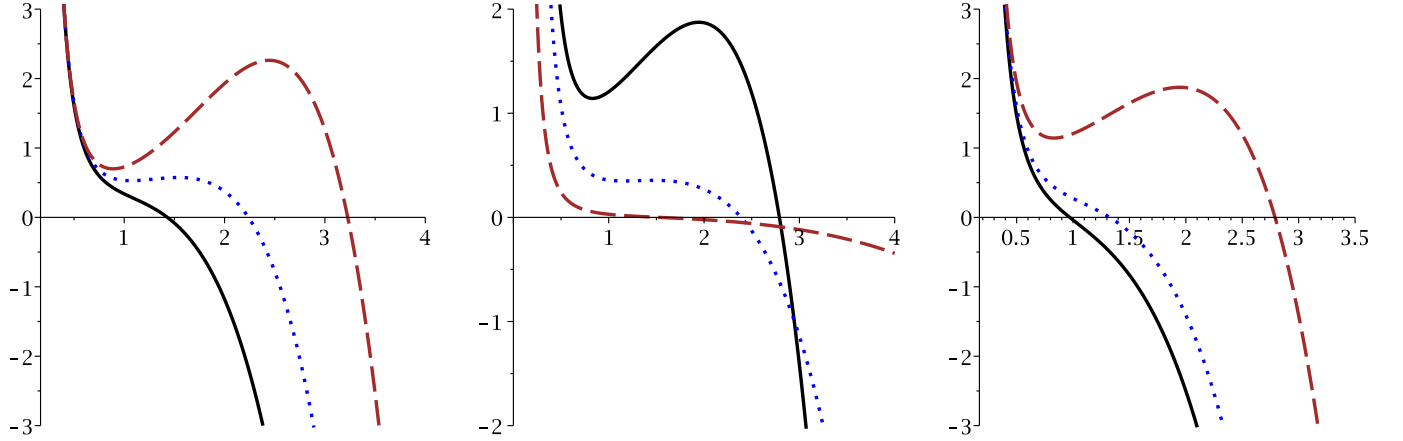


FIG. 4: $\psi(r)$ versus r for $q = 1$, $c = c_1 = c_2 = c_3 = c_4 = 0.1$, $d = 6$ and $\Lambda = 1$.

Left diagram for $g(E) = f(E) = 0.8$, $m = 1.1$, $k = -1$ (continues line), $k = 0$ (dotted line) and $k = 1$ (dashed line).

Middle diagram for $k = -1$, $m = 2$, $g(E) = f(E) = 0.8$ (continues line), $g(E) = f(E) = 1$ (dotted line) and $g(E) = f(E) = 2$ (dashed line).

Right diagram for $k = -1$, $g(E) = f(E) = 0.8$, $m = 0$ (continues line), $m = 11$ (dotted line) and $m = 2$ (dashed line).

which are the same as the ones calculated for the temperature and the electric potential which are calculated in Eqs. (26) and (31). As a result, the first law of thermodynamics is valid while some of quantities modified in the presence of the massive term and rainbow functions.

Our final study is regarding the behavior of the mass total mass. In obtained relation for total mass of the black holes (Eq. 30) for positive mass coefficients, the only factors which contribute to negativity and positivity of the mass are cosmological and topological terms. In case of anti de Sitter solutions ($\Lambda = -1$), for spherical and flat horizons, mass of the black holes are always positive and for specific values there may be a minimum for mass. In these cases, the black holes size and mass can expand without any limitation. As for hyperbolic horizon ($k = -1$), by suitable choices of different parameter, mass may acquire one extreme root. In this case too, black holes size could grow without any limitation. By considering another set of values for different parameters, one may encounter two roots for mass with a region of negativity. In this case, the black holes with positive mass are limited to two regions. Then again, for this case, except for negative region, black holes' size could grow without any limitation. One can see that minimum, hence, number of the roots, are increasing (decreasing) functions of energy functions (mass term) (see middle and right panels of Fig. 3).

In case of de Sitter spacetime, black holes radius could not grow without any limitation (Fig. 3). In this case, depending on the choices of different parameters, mass could be a decreasing function of the horizon with a root or it may form two extrema with one root. In both of these cases, the size of black holes are limited. In other words, positive valued mass only exists for a range of horizon radius and the black holes are restricted to exist in specific region. It is evident that number of the extrema and root are increasing (decreasing) functions of energy functions (mass term) (see middle and right panels of Fig. 4).

V. CLOSING REMARKS

In this paper, we have considered massive gravity in an energy dependent spacetime. At first we obtain exact charged black hole solutions and investigate their geometric properties. It was shown that although considering this configuration leads to modification of the number and place of horizons that black holes can acquire, the asymptotical adS behavior of the solutions has no change. We relax the phenomenologies of the multiple horizons and focused on the thermodynamical properties of the black holes at the outer horizon. We obtained conserved and thermodynamic quantities and their modifications are observed. Then we checked the validity of the first law of thermodynamics and found that it holds for the obtained black holes.

Next, we conducted a study regarding the behavior of the mass term in obtained solutions. In this regard, we highlighted the differences between black holes in de Sitter and anti de Sitter spacetime. We showed that black holes in anti de Sitter spacetime could grow without any limitation except for a region which is observed only in hyperbolic horizon. While in de Sitter spacetime, the positive valued mass only existed for specific range and the black holes in case of these solutions could not grow without any limitation.

Acknowledgments

We thank Shiraz University Research Council. This work has been supported financially by the Research Institute for Astronomy and Astrophysics of Maragha, Iran.

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